An Overview of Abstract Interpretation and Program Static Analysis

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Motivations
What is (or should be) the main preoccupation of computer scientists?
What is (or should be) the main preoccupation of computer scientists?

The production of reliable software, its maintenance and safe evolution year after year (up to 20 to 30 years).
Computer hardware change of scale

The 25 last years, computer hardware has seen its performances multiplied by $10^4$ to $10^6$;
The information processing revolution

A scale of $10^6$ is typical of a significant **revolution**:
- **Energy**: nuclear power station / Roman slave;
- **Transportation**: distance Earth — Mars / height of Korea
Computer software change of scale

- The size of the programs executed by these computers has grown up in similar proportions;
Computer software change of scale

- The size of the programs executed by these computers has grown up in similar proportions;
- **Example 1** (modern text editor for the general public):
  - > 1 700 000 lines of C
  - 20 000 procedures;
  - 400 files;
  - > 15 years of development.

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\(^2\) full-time reading of the code (35 hours/week) would take at least 3 months!
Computer software change of scale (cont’d)

- Example 2 (professional computer system):
  - 30,000,000 lines of code;
Computer software change of scale (cont’d)

• **Example 2** (professional computer system):
  - 30 000 000 lines of code;
  - 30 000 (known) bugs!
Bugs

- **Software bugs**
  - whether anticipated (Y2K bug)
  - or unforeseen (failure of the 5.01 flight of Ariane V launcher)

are quite frequent;
Bugs

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are quite frequent;

- Bugs can be very **difficult to discover** in huge software;
Bugs

• Software bugs
  – whether anticipated (Y2K bug)
  – or unforeseen (failure of the 5.01 flight of Ariane V launcher) are frequent;

• Bugs can be very difficult to discover in huge software;

• Bugs can have catastrophic consequences either very costly or inadmissible (embedded software in transportation systems);
The estimated cost of an overflow
The estimated cost of an overflow

- $ 500 000 000
The estimated cost of an overflow

- $500,000,000
- Including indirect costs (delays, lost markets, etc):
  - $2,000,000,000,000
The intellectual capability of computer scientists remains essentially unchanged year after year;
Capability of computer scientists

- The intellectual capability of computer scientists remains essentially unchanged year after year;

- The size of programmer teams in charge of software design and maintenance cannot evolve in such huge proportions;
Capability of computer scientists

- The intellectual capability of computer scientists remains essentially unchanged year after year;

- The size of programmer teams in charge of software design and maintenance cannot evolve in such huge proportions;

- Classical manual software verification methods (code reviews, simulations, debugging) do not scale up.
Responsibility of computer scientists

- The **paradox** is that the computer scientists do not assume any **responsibility** for software bugs (compare to the automotive or avionic industry);
Responsibility of computer scientists

• The paradox is that the computer scientists do not assume any responsibility for software bugs (compare to the automotive or avionic industry);

• Computer software bugs can become an important societal problem (collective fears and reactions? new legislation?);
Responsibility of computer scientists

- The **paradox** is that the computer scientists do not assume any responsibility for software bugs (compare to the automotive or avionic industry);
- Computer software bugs can become an important **societal problem** (collective fears and reactions? new legislation?);
- The combat against software bugs might even be the next **worldwide war**;
Responsibility of computer scientists

• The **paradox** is that the computer scientists do not assume any responsibility for software bugs (compare to the automotive or avionic industry);

• Computer software bugs can become an important **societal problem** (collective fears and reactions? new legislation?)

• The combat against software bugs might even be the next worldwide war;

$$\Rightarrow$$ It is absolutely necessary to widen the full set of methods and tools used to fight against software bugs.
Idea

Use the computer to find programming errors.
(Extremely difficult) question

How can computers be programmed so as to analyze the work they are given to do before effectively doing it?
A simplistic example: a cooking recipe

The soft-boiled egg recipe:

- Take a fresh egg out of the refrigerator;
- Plunged it into salted boiling water;
- Pull it out of the water after 4 mn.
A simplistic example: a cooking recipe

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Why not computers?
A simplistic example: a cooking recipe

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- Plunged it into salted boiling water;
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Any cook can find the bug before carrying out the recipe!

Why not computers?

What can we do about it?
Considered approaches for program verification
Considered approaches for program verification

Deductive methods
Considered approaches for program verification

**Deductive methods:** The proof size is exponential in the program size!
Considered approaches for program verification

Deductive methods: The proof size is exponential in the program size!

Model-checking
Considered approaches for program verification

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**Model-checking:** Gained only a factor of 100 in 10 years and the limit seems to be reached!
Considered approaches for program verification

**Deductive methods:** The proof size is exponential in the program size!

**Model-checking:** Gained only a factor of 100 in 10 years and the limit seems to be reached!

**What else?**
Introductory Talk

- Four notions to be introduced:
  - Semantics,
  - Undecidability,
  - Abstract interpretation,
  - Program static analysis;
Informal Introductory Talk

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  - Semantics,
  - Undecidability,
  - Abstract interpretation,
  - Program static analysis;
- Completely informal explanation avoiding any formalism;
Informal Introductory Talk

• Four notions to be introduced:
  – Semantics,
  – Undecidability,
  – Abstract interpretation,
  – Program static analysis;

• Completely informal explanation avoiding any formalism;

• Illustrated by the work done in my research team and the theses that I directed since 10 years.
Semantics & Undecidability
Hence we must first explain semantics, for example:

**Syntax:**

\[ x, f \in X \quad : \text{variables} \]
\[ e \in E \quad : \text{expressions} \]
\[ e ::= x \mid \lambda x \cdot e \mid e_1(e_2) \mid \mu f \cdot \lambda x \cdot e \mid e_1 - e_2 \mid 1 \mid (e_1 ? e_2 : e_3) \]

**Semantic domains:**

\[ W \triangleq \{ \omega \} \quad \text{error} \]
\[ z \in Z \quad \cong W \perp \oplus Z \perp \oplus [U \mapsto U] \perp \quad \text{integers} \]
\[ u, f, \varphi \in U \quad \cong W \perp \oplus Z \perp \oplus [U \mapsto U] \perp \quad \text{values} \]
\[ R \in R \quad \triangleq X \mapsto U \quad \text{environments} \]
\[ \phi \in S \quad \triangleq R \mapsto U \quad \text{semantic domain} \]

**Semantics:**

\[
\begin{align*}
S[x] & \triangleq \Lambda R \cdot R(x) \\
S[\lambda x \cdot e] & \triangleq \Lambda R \cdot \top (\Lambda u \cdot (u = \bot \lor u = \Omega ? u | S[e]R[x \leftarrow u]) :: [U \mapsto U] \perp) \\
S[e_1(e_2)] & \triangleq \Lambda R \cdot \top (\Lambda f \cdot \top (\Lambda e_1 \cdot \top (\Lambda e_2 \cdot \top (\Lambda f \cdot \top (\Lambda \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi]))) :: [U \mapsto U] \perp) \\
S[e_1 - e_2] & \triangleq \Lambda R \cdot \top (\Lambda e_1 \cdot \top (\Lambda e_2 \cdot \top (\Lambda f \cdot \top (\Lambda \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi]))) :: [U \mapsto U] \perp) \\
S[(e_1 ? e_2 : e_3)] & \triangleq \Lambda R \cdot \top (\Lambda e_1 \cdot \top (\Lambda e_2 \cdot \top (\Lambda f \cdot \top (\Lambda \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi]))) :: [U \mapsto U] \perp) \\
\end{align*}
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Semantics:

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\begin{align*}
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S[\lambda x \cdot e] & \triangleq \Lambda R \cdot (\Lambda u \cdot (u = \perp \lor u = \Omega ? u \mid \\
& S[e][R[x \leftarrow u]])) :: [U \mapsto U] \_ \\
S[e_1(e_2)] & \triangleq \Lambda R \cdot (S[e_1]R = \perp \lor S[e_2]R = \perp ? \perp \mid \\
& S[e_1]R = f :: [U \mapsto U] \_ ? \uparrow(f)(S[e_2]R) | \Omega) \\
S[\mu f \cdot \lambda x \cdot e] & \triangleq \Lambda R \cdot \text{lfp}^{C_{(U \downarrow) \cup [U \mapsto U] \_ \downarrow}} \Lambda \varphi \cdot S[\lambda x \cdot e][f \leftarrow \varphi] \\
S[1] & \triangleq \Lambda R \cdot \uparrow(1) :: Z \_ \\
S[e_1 - e_2] & \triangleq \Lambda R \cdot (S[e_1]R = \perp \lor S[e_2]R = \perp ? \perp \mid \\
& S[e_1]R = z_1 :: Z \_ \land S[e_2]R = z_2 :: Z \_ \mid \\
& \uparrow(\downarrow(z_1) - \downarrow(z_2)) :: Z \_ | \Omega) \\
S[(e_1 ? e_2 : e_3)] & \triangleq \Lambda R \cdot (S[e_1]R = \perp ? \perp \mid S[e_1]R = z :: Z \_ ? \\
& (\downarrow(z) = 0 ? S[e_2]R | S[e_3]R) | \Omega)
\end{align*}
\]
Hence we must first explain semantics, for example:

**Syntax:**

- \(x, f \in X\) : variables
- \(e \in E\) : expressions
  - \(e \ ::= x | \lambda x \cdot e | e_1(e_2) | \mu f \cdot \lambda x \cdot e | e_1 - e_2 | 1 | (e_1 ? e_2 : e_3)\)

**Semantic domains:**

- \(W \triangleq \{\omega\}\) : error
- \(z \in Z\) : integers
- \(u, f, \varphi \in U\) : \(W \perp \oplus Z \perp \oplus [U \mapsto U] \perp\) values
- \(R \in R\) : \(X \mapsto U\) : environments
- \(\phi \in S\) : \(R \mapsto U\) : semantic domain

**Semantics:**

- \(S[x] \triangleq \lambda R \cdot R(x)\)
- \(S[\lambda x \cdot e] \triangleq \Lambda R \cdot ((\Lambda u \cdot (u = \bot \lor u = \Omega) ? u | S[e]R[x \leftarrow u]) :: [U \mapsto U] \perp)\)
- \(S[e_1(e_2)] \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot ? \bot | S[e_1]R = f :: [U \mapsto U] \perp ? \bot | S[e_2]R ? \bot | \Omega)\)
- \(S[\mu f \cdot \lambda x \cdot e] \triangleq \Lambda R \cdot \text{lfp} \downarrow_{\text{\{}(\Lambda u \cdot \bot, u = \Omega)\}} \Lambda \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi]\)
- \(S[1] \triangleq \lambda R \cdot 1(1) :: Z \perp\)
- \(S[e_1 - e_2] \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot ? \bot | S[e_1]R = z_1 :: Z \perp \lor S[e_2]R = z_2 :: Z \perp ? \bot | S[e_1]R = z_1 :: Z \perp \lor S[e_2]R = z_2 :: Z \perp ? \bot | \Omega)\)
- \(S[(e_1 ? e_2 : e_3)] \triangleq \Lambda R \cdot (S[e_1]R = \bot ? \bot | S[e_1]R = z :: Z \perp ? \bot | S[e_2]R \lor S[e_3]R ? \bot | \Omega)\)

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Hence we must first explain semantics, for example:

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**Semantics:**

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S[x] \triangleq \Lambda R \cdot R(x) \\
S[\lambda x \cdot e] \triangleq \Lambda R \cdot \uparrow (\Lambda u \cdot (u = \bot \lor u = \Omega) \mid S[e]R[\langle x \mapsto u \rangle]) :: [U \mapsto \bot] \\
S[e_1(e_2)] \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot) \mid \\
\text{if} :: [U \mapsto \bot] \mid \downarrow (\varphi) (S[e_2]R | \Omega) \\
S[\mu f \cdot \lambda x \cdot e] \triangleq \Lambda R \cdot \text{lfp}_{((\Lambda u \cdot \bot) : [U \mapsto \bot])} \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi] \\
S[1] \triangleq \Lambda R \cdot \uparrow (1) :: Z \bot \\
S[e_1 - e_2] \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot) \mid \\
\text{if} :: [U \mapsto \bot] \mid \downarrow (\varphi) (S[z_1 \mapsto Z \bot \land S[e_2]R = z_2 :: Z \bot) \mid \\
\downarrow (\varphi) (S[z_1 \mapsto Z \bot \land S[e_2]R = z_2 :: Z \bot) \mid \Omega) \\
S[(e_1 ? e_2 : e_3)] \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = z :: Z \bot) \mid \\
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Semantics

- The semantics of a program provides a formal mathematical model of all possible behaviors of a computer system executing this program (interacting with any possible environment);
Semantics

- The **semantics of a program** provides a **formal mathematical model of all possible behaviors** of a computer system executing this program (interacting with any possible environment);

- The **semantics of a language** defines the semantics of any program written in this language.
Example 1: trace semantics

Initial states

Intermediate states

Final states of the finite traces

Infinite traces

Discrete time

0 1 2 3 4 5 6 7 8 9

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Examples of computation traces

• Finite \((C1+1=)\):

• Erroneous \((C1+1+1+1...)\):

• Infinite \((C+0+0+0...)\):
Example 2: geometric semantics

\[
\llbracket \ Pa.Pb.Va.Vb \\
\parallel Pb.Pc.Vb.Vc \\
\parallel Pc.Pa.Vc.Va \rrbracket
\]

É. Goubault thesis, 1995
Example 2: geometric semantics (deadlock)

\[
| Pb.Pc.Vb.Vc \\
\]

deadlock

É. Goubault thesis, 1995
Undecidability

• All interesting questions relative to the semantics of non-trivial programs are undecidable; ⇒ no computer can always exactly answer such questions in finite time;

• One can mathematically define the semantics of a program as the solution of a fixpoint equation ⇒ but no computer can exactly solve these equations in finite time.
Undecidability

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Undecidability

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  - no computer can always exactly answer such questions in finite time;
- One can mathematically define the semantics of a program as the solution of a fixpoint equation;
Undecidability

- All interesting questions relative to the semantics of non trivial programs are **undecidable**:
  \[ \Rightarrow \text{no computer can always exactly answer such questions in finite time}; \]
- One can mathematically define the semantics of a program as the solution of a fixpoint equation:
  \[ \Rightarrow \text{but no computer can exactly solve these equations in finite time.} \]
Semantics and fixpoints

Syntax:

\[ \begin{align*}
  x, f & \in X : \text{variables} \\
  e & \in E : \text{expressions} \\
  e & ::= x \mid \lambda x \cdot e \mid e_1(e_2) \mid \\
  & \mu f \cdot \lambda x \cdot e \mid e_1 - e_2 \mid 1 \mid (e_1 ? e_2 : e_3)
\end{align*} \]

Semantic domains:

\[ \begin{align*}
  W & \triangleq \{ \omega \} : \text{error} \\
  Z & \in \mathbb{Z} : \text{integers} \\
  U, f, \varphi & \in U : \text{values} \\
  R & \in R : \text{environments} \\
  S & \in S : \text{semantic domain}
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Semantics:

\[ \begin{align*}
  S[x] & \triangleq \Lambda R \cdot R(x) \\
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  & S[e]R[x \leftarrow u]) : [U \mapsto U]_\bot) \\
  S[e_1(e_2)] & \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot ? \bot | \\
  & S[e_1]R = f : [U \mapsto U]_\bot ? \downarrow (f)(S[e_2]R) | \Omega) \\
  S[\mu f \cdot \lambda x \cdot e] & \triangleq \Lambda R \cdot \text{lfp}_{\bot \leq \uparrow} \Lambda \varphi \cdot S[\lambda x \cdot e]R[f \leftarrow \varphi] \\
  S[1] & \triangleq \Lambda R \cdot \uparrow (1) : Z_\bot \\
  S[e_1 - e_2] & \triangleq \Lambda R \cdot (S[e_1]R = \bot \lor S[e_2]R = \bot ? \bot | \\
  & S[e_1]R = z_1 : Z_\bot \land S[e_2]R = z_2 : Z_\bot ? \uparrow (\downarrow (z_1) - \downarrow (z_2)) : Z_\bot | \Omega) \\
  S[(e_1 ? e_2 : e_3)] & \triangleq \Lambda R \cdot (S[e_1]R = \bot ? \bot | S[e_1]R = z : Z_\bot ? \uparrow (\downarrow (z) = 0 ? S[e_2]R | S[e_3]R) | \Omega)
\end{align*} \]
Least Fixpoints: Intuition

Behaviors =

In general, the equation has multiple solutions. Choose the least one for the partial ordering: « more finite traces & less infinite traces ».
Fixpoints: Intuition

Behaviors = \{ • \mid • is a final state \}
Least Fixpoints: Intuition

Behaviors = \{ \bullet \ | \ \bullet \ is \ a \ final \ state \} \\
\qquad \cup \ \{ \bullet \ldots \bullet \ | \ \bullet \ldots \bullet \ is \ an \ elementary \ step \ & \ \bullet \ldots \bullet \in \ Behaviors \}
Least Fixpoints: Intuition

\[ \text{Behaviors} = \{ \bullet \mid \bullet \text{ is a final state} \} \]
\[ \cup \{ \bullet \cdots \bullet \mid \bullet \cdots \bullet \text{ is an elementary step } \in \text{Behaviors} \} \]
\[ \cup \{ \bullet \cdots \bullet \cdots \cdots \mid \bullet \cdots \bullet \text{ is an elementary step } \in \text{Behaviors}^{\infty} \} \]

In general, the equation has multiple solutions. Choose the least one for the partial ordering: « more finite traces & less infinite traces ». 

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Fixpoints: Intuition

Behaviors = \{ • \mid • \text{ is a final state}\}
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• \rightarrow \cdots • \in \text{Behaviors}\} \\
\bigcup \{ • \rightarrow \cdots • \rightarrow \cdots • \mid • \rightarrow • \text{ is an elementary step} \&
• \rightarrow \cdots • \rightarrow \cdots • \in \text{Behaviors}^{\infty}\}

In general, the equation has multiple solutions.
Least Fixpoints: Intuition

Behaviors = \{ \bullet | \bullet \text{ is a final state} \} \
\bigcup \{ \bullet \ldots \bullet | \bullet \ldots \bullet \text{ is an elementary step &} \\
\bullet \ldots \bullet \in \text{Behaviors} \} \
\bigcup \{ \bullet \ldots \bullet | \bullet \ldots \bullet \text{ is an elementary step &} \\
\bullet \ldots \bullet \in \text{Behaviors}^\infty \}

In general, the equation has multiple solutions. Choose the least one for the partial ordering:

« more finite traces & less infinite traces ». 
Abstract Interpretation
Abstract interpretation

• Abstract interpretation is a theory of the approximation of the behavior of discrete systems, including the semantics of (programming or specification) languages;
Abstract interpretation

- Abstract interpretation is a theory of the approximation of the behavior of discrete systems, including the semantics of (programming or specification) languages;

- Abstract interpretation formalizes the intuitive idea that a semantics is more or less precise according to the considered observation level.
### Familiar abstraction examples

<table>
<thead>
<tr>
<th>concrete</th>
<th>abstract</th>
</tr>
</thead>
<tbody>
<tr>
<td>citizen</td>
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</tr>
<tr>
<td>road network</td>
<td></td>
</tr>
<tr>
<td>film</td>
<td></td>
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<td>car</td>
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<tr>
<td>scientific article</td>
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<tr>
<td>scientific article</td>
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<td>number</td>
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## Familiar abstraction examples

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</tr>
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<td>car</td>
<td>trade mark</td>
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<tr>
<td>scientific article</td>
<td>abstract</td>
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<tr>
<td>scientific article</td>
<td>keywords</td>
</tr>
<tr>
<td>number</td>
<td>sign and/or parity</td>
</tr>
</tbody>
</table>
Examples of approximate semantics

\[ \begin{array}{c}
\text{Initial states} \\
\text{Intermediate states} \\
\text{Final states of finite traces}
\end{array} \]

\[ \begin{array}{c}
\{a, b, c, d\} \\
\{e, f, g, h\} \\
\{i, j, k, \ell\}
\end{array} \]

\[ \begin{array}{c}
0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \\
discrete time
\end{array} \]

Trace semantics

\[ \begin{array}{c}
\text{Initial states} \\
\{a, d\} \\
\{e, f\} \\
\{g, h\} \\
\{i, j\} \\
\{k, \perp\} \\
\{\ell, \perp\}
\end{array} \]

Denotational semantics

\[ \alpha \]

Natural semantics

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Information loss

- Because of the information loss, not all questions can be definitely answered;
Information loss

• Because of the information loss, not all questions can be definitely answered;

• All answers given by the abstract semantics are always correct with respect to the concrete semantics.
# Example of information loss

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Abstract</th>
</tr>
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<tbody>
<tr>
<td>trace semantics</td>
<td>denotational semantics</td>
</tr>
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</table>

**Question**

Starting from state $g$, can execution terminate in state $h$?

Does execution starting from state $k$ always terminate? **no**

Can state $b$ be immediately followed by state $c$? **yes**
Semantics

Trace semantics

Initial states
Intermediate states
Final states of finite traces

Discrete time

Infinite traces

Initial states
Final states of
finite traces

Denotational semantics

Natural semantics

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**Example of information loss**

<table>
<thead>
<tr>
<th>Question</th>
<th>Concrete ←</th>
<th>denotational semantics</th>
<th>natural semantics</th>
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The more concrete semantics can answer more questions. The more abstract semantics are more simple.
## Example of information loss

<table>
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<tr>
<th>Concrete</th>
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<tbody>
<tr>
<td><strong>Question</strong></td>
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</tbody>
</table>
Semantics

Trace semantics

Initial states
Intermediate states
Final states of finite traces

Infinite traces

Discrete time

Denotational semantics
Natural semantics

Initial states
Final states

0 1 2 3 4 5 6 7 8 9
### Example of information loss

<table>
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<th>Abstract natural semantics</th>
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The more concrete semantics can answer more questions. The more abstract semantics are more simple.
Example of non comparable approximated semantics

Initial states

\[
\begin{align*}
\{ & a, e, g, i, k, \ell \} \\
\end{align*}
\]

Transitions

\[
\begin{align*}
\{ & a, b, c, d, f, g, h, i, j \} \\
\end{align*}
\]

Final states

\[
\{ & d, f, h, j \} \\
\end{align*}
\]

Operational semantics

---


---

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What is the information loss?

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Operational semantics

Initial states

\[
\{a, e, g, i, k, \ell\}
\]

Transitions

\[
\{a, b, c, d, f, h, j\}
\]

Final states

\[
\{d, f, h, j\}
\]
The information loss is incomparable

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Computable approximations

- If the approximation is rough enough, the abstraction of a semantics can lead to a version which is less precise but is effectively computable by a computer;
Computable approximations

- If the approximation is rough enough, the abstraction of a semantics can lead to a version which is less precise but is effectively computable by a computer;

- By effective computation of the abstract semantics, the computer is able to analyze the behavior of programs and of software before and without executing them.
Example of computable approximations of an [in]finite set of points

\( \{ \ldots, \langle 19, 88 \rangle, \ldots, \langle 19, 99 \rangle, \ldots \} \)
Example of computable approximations of an \([\text{in}]\)finite set of points (signs)

\[
\begin{cases}
x \geq 0 \\
y \geq 0
\end{cases}
\]
Example of computable approximations of an \([\text{in}]\)finite set of points (intervals)

\[
\begin{aligned}
x &\in [19, 88] \\
y &\in [19, 99]
\end{aligned}
\]
Example of computable approximations of an [in]finite set of points (octagons)

\[
\begin{align*}
1 & \leq x \leq 9 \\
x + y & \leq 88 \\
1 & \leq y \leq 9 \\
x - y & \leq 99
\end{align*}
\]
Example of computable approximations of an \([\text{in}]\)finite set of points (polyhedra)

\[
\begin{align*}
19x + 88y & \leq 2000 \\
19x + 99y & \geq 0 
\end{align*}
\]

P. Cousot & N. Halbwachs, POPL’78
Example of computable approximations of an [in]finite set of points (simple congruences)

\[
\begin{cases}
x = 19 \mod 88 \\
y = 19 \mod 99
\end{cases}
\]
Example of computable approximations of an [in]finite set of points (linear congruences)

\[
\begin{align*}
1x + 9y &= 8 \mod 8 \\
1x - 9y &= 9 \mod 9
\end{align*}
\]
Example of computable approximations of an [in]finite set of points (trapezoidal linear congruences)

\[
\begin{align*}
1x + 9y & \in [0, 88] \text{ mod } 10 \\
1x - 9y & \in [0, 99] \text{ mod } 11
\end{align*}
\]
Application of the congruence analysis: communications in OCCAM

Communications of "i/o buffer"

thesis N. Mercouroff, 1990

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More difficult: non numerical structures

• Most structures manipulated by programs are not numerical (so called symbolic structures);
• It is the case, for example, of the following structures:
  -- control structures (call graphs, recursion trees),
  -- data structures (search trees),
  -- communication structures (distributed programs),
  -- information transfer structures (mobile programs), etc.
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Example 1: (infinite) sets of (infinite) decorated trees
Example 2: (infinite) set of (infinite) decorated graphs
Precise compact approximations
Precise compact approximations

• It is very difficult to find compact and expressive computer representations of such sets of objects (languages, automata, trees, graphs, etc.)
Precise compact approximations

• It is very difficult to find compact and expressive computer representations of such sets of objects (languages, automata, trees, graphs, etc.) such that:
  – the various set-theoretic operations can be efficiently implemented;
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Example of compact approximations of infinite sets of infinite trees

Binary Decision Graphs:

Tree schemata:

Note that $E$ is the equality relation.

these L. Mauborgne, 1999
Program Static Analysis
Difficulty of programming

- Large scale computer programming is very difficult;
Difficulty of programming

- Large scale computer programming is very difficult;
- Reasoning on large programs is very difficult;
Difficulty of programming

- Large scale computer programming is very difficult;
- Reasoning on large programs is very difficult;
- Errors are quite frequent.
Example 1: first year exam at the École polytechnique

What is the effect of the following PASCAL program:

```pascal
program P (input, output);
procedure NewLine; begin writeln end;
procedure P (X : integer; procedure Q);
procedure R;
begin write(X); Q; end;
begin
if X > 0 then begin R; P(X - 1, R); end;
end;
begin
P(5, NewLine);
end.
```

Less than 5% of the answers are correct!
Example 1: first year exam at the École polytechnique

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program P (input, output);
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  P(5, NewLine);
end.
```

Less than 5% of the answers are correct!
Example 2: first year exam at the École polytechnique

Prove that the following program prints the value $\geq 91$:

```pascal
program MacCarthy (input, output);
    var x, m : integer;
    function MC(n : integer) : integer;
        begin
            if n > 100 then MC := n - 10
            else MC := MC(MC(n + 11));
        end;
    begin
        read(x); m := MC(x); writeln(m);
    end.
```

Less than 50% of the proofs given as answers are correct!
Example 2: first year exam at the École polytechnique

Prove that the following program prints the value $\geq 91$:

```pascal
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var x, m : integer;
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Less than 50 % of the proofs given as answers are correct!
Program static analysis

- **Objective:** discover programming errors before they lead to disastrous catastrophes!

Program static analysis uses abstract interpretation to derive, from a standard semantics, an approximate and computable semantics; it follows that the computer is able to analyze the behavior of software before and without executing it; this is essential for computer-based safety-critical systems (for example: planes, trains, launchers, nuclear plants, etc.).
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• It follows that the computer is able to analyze the behavior of software before and without executing it;

• This is essential for computer-based safety-critical systems (for example: planes, trains, launchers, nuclear plants, etc.).
Example: interval analysis (1975)

Program to be analyzed:

\[
\begin{align*}
x & := 1; \\
1: & \quad \text{while } x < 10000 \text{ do} \\
2: & \quad x := x + 1 \\
3: & \quad \text{od;}
\end{align*}
\]

\[X_1 = [1, 1]
\]
\[X_2 = (X_1 \cup X_3) \cap [\neg \infty, 9999]
\]
\[X_3 = X_2 \oplus [1, 1]
\]
\[X_4 = (X_1 \cup X_3) \cap [10000, \infty]
\]
Example: interval analysis (1975)  

Equations (abstract interpretation of the semantics):

\[
\begin{align*}
x & := 1; \\
1: & \quad \text{while } x < 10000 \text{ do} \\
2: & \quad x := x + 1 \\
3: & \quad \text{od}; \\
4: & \\
\end{align*}
\]

\[
\begin{align*}
X_1 & = [1, 1] \\
X_2 & = (X_1 \cup X_3) \cap [-\infty, 9999] \\
X_3 & = X_2 \oplus [1, 1] \\
X_4 & = (X_1 \cup X_3) \cap [10000, +\infty]
\end{align*}
\]

---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)

Increasing chaotic iteration, initialization:

\[
\begin{align*}
x &:= 1; \\
\text{while } x < 10000 \text{ do} & \\
x &:= x + 1 \\
\text{od}; \\
\end{align*}
\]

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\begin{align*}
X_1 &= [1, 1] \\
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\end{align*}
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Increasing chaotic iteration:

```
x := 1;
while x < 10000 do
  x := x + 1
od;
```

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---

5 P. Cousot & R. Cousot, ISOP'1976, POPL'77.

---

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Example: interval analysis (1975) \(^5\)

Increasing chaotic iteration:

\[
\begin{align*}
  x &:= 1; \\
  1: & \quad \text{while } x < 10000 \text{ do} \\
  2: & \quad x := x + 1 \\
  3: & \quad \text{od;} \\
  4: & \quad 
\end{align*}
\]

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\begin{align*}
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\end{align*}
\]

\[
\begin{align*}
  X_1 &= [1, 1] \\
  X_2 &= [1, 1] \\
  X_3 &= \emptyset \\
  X_4 &= \emptyset
\end{align*}
\]

\(^5\) P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975) \(^5\)

Increasing chaotic iteration:

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\begin{align*}
x & := 1; \\
\text{1:} & \quad \text{while } x < 10000 \text{ do} \\
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\text{3:} & \quad \text{od;} \\
\text{4:} & \quad \end{align*}
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\begin{align*}
X_1 &= [1, 1] \\
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X_3 &= X_2 \oplus [1, 1] \\
X_4 &= (X_1 \cup X_3) \cap [10000, +\infty]
\end{align*}
\]

\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 1] \\
X_3 &= [2, 2] \\
X_4 &= \emptyset
\end{align*}
\]

\(^5\) P. Cousot & R. Cousot, ISOP'1976, POPL'77.
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Increasing chaotic iteration:

\[ x := 1; \]

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\[ \textbf{od}; \]

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5 P. Cousot & R. Cousot, ISOP'1976, POPL'77.
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Increasing chaotic iteration: convergence?

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x &:= 1; \\
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\end{align*}
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\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 2] \\
X_3 &= [2, 3] \\
X_4 &= \emptyset
\end{align*}
\]

---

5 P. Cousot & R. Cousot, ISOP'1976, POPL'77.
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\]

\[
\begin{aligned}
X_1 &= [1, 1] \\
X_2 &= [1, 3] \\
X_3 &= [2, 3] \\
X_4 &= \emptyset
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---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)  
Increasing chaotic iteration: convergence???

\[ x := 1; \]
\[ \textbf{1:} \]
\[ \textbf{while } x < 10000 \textbf{ do} \]
\[ \textbf{2:} \]
\[ x := x + 1 \]
\[ \textbf{3:} \]
\[ \textbf{od;} \]
\[ \textbf{4:} \]

\[
\begin{aligned}
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\end{aligned}
\]

\[
\begin{aligned}
X_1 &= [1, 1] \\
X_2 &= [1, 3] \\
X_3 &= [2, 4] \\
X_4 &= \emptyset
\end{aligned}
\]

---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.

---

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\end{align*}
\]

\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 4] \\
X_3 &= [2, 4] \\
X_4 &= \emptyset
\end{align*}
\]

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)

Increasing chaotic iteration: convergence????

\[
x := 1;
\]

1:
while \( x < 10000 \) do

2:
\[
x := x + 1
\]

3:
\[\od\]

4:
\[
\begin{align*}
X_1 &= [1, 1] \\
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\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 4] \\
X_3 &= [2, 5] \\
X_4 &= \emptyset
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\[5\] P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)  

Increasing chaotic iteration: convergence??????
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\end{align*}
\]

\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 5] \\
X_3 &= [2, 6] \\
X_4 &= \emptyset
\end{align*}
\]

---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)

Convergence speed-up by extrapolation:

\[ x := 1; \]
\[ \text{while } x < 10000 \text{ do} \]
\[ \quad x := x + 1 \]
\[ \text{od}; \]

\[
\begin{align*}
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X_2 &= (X_1 \cup X_3) \cap [-\infty, 9999] \\
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\]
Example: interval analysis (1975)

Decreasing chaotic iteration:

\[
\begin{align*}
x & := 1; \\
1: & \\
& \text{while } x < 10000 \text{ do} \\
2: & \quad x := x + 1 \\
3: & \quad \text{od;} \\
4: &
\end{align*}
\]

\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= (X_1 \cup X_3) \cap [-\infty, 9999] \\
X_3 &= X_2 \oplus [1, 1] \\
X_4 &= (X_1 \cup X_3) \cap [10000, +\infty]
\end{align*}
\]

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975) \(^5\)

Decreasing chaotic iteration:

\[
\begin{align*}
x &:= 1; \\
1: & \quad \text{while } x < 10000 \text{ do} \\
2: & \quad x := x + 1 \\
3: & \quad \text{od;}
\end{align*}
\]

\[
\left\{ 
\begin{aligned}
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\end{aligned}
\right.
\]

\[
\left\{ 
\begin{aligned}
X_1 &= [1, 1] \\
X_2 &= [1, 9999] \\
X_3 &= [2, +\infty] \\
X_4 &= \emptyset
\end{aligned}
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\(^5\) P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)

Decreasing chaotic iteration:

\begin{align*}
x & := 1; \\
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\begin{cases}
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---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.

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Example: interval analysis (1975)

Final solution:

```
x := 1;
1:  while x < 10000 do
2:    x := x + 1
3:  od;
4:
```

\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= (X_1 \cup X_3) \cap [-\infty, 9999] \\
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\[
\begin{align*}
X_1 &= [1, 1] \\
X_2 &= [1, 9999] \\
X_3 &= [2, +10000] \\
X_4 &= [+10000, +10000]
\end{align*}
\]
Example: interval analysis (1975)

Result of the interval analysis:

\[ x := 1; \]
\[ 1: \{x = 1\} \]
\[ \text{while } x < 10000 \text{ do} \]
\[ 2: \{x \in [1, 9999]\} \]
\[ \quad x := x + 1 \]
\[ 3: \{x \in [2, +10000]\} \]
\[ \text{od;} \]
\[ 4: \{x = 10000\} \]

\[
\begin{align*}
X_1 &= [1, 1] \\
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\begin{align*}
X_1 &= [1, 1] \\
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X_4 &= [+10000, +10000]
\end{align*}
\]

---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.
Example: interval analysis (1975)  

Exploitation of the result of the interval analysis:

\[
\begin{align*}
x &:= 1; \\
1: & \{ x = 1 \} \\
& \text{while } x < 10000 \text{ do} \\
2: & \{ x \in [1, 9999] \} \\
& \quad x := x + 1 \\
3: & \{ x \in [2, +10000] \} \\
& \quad \text{od; } \\
4: & \{ x = 10000 \}
\end{align*}
\]

\[\begin{align*}
X_1 &= [1, 1] \\
X_2 &= (X_1 \cup X_3) \cap [1, 9999] \\
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X_4 &= (X_1 \cup X_3) \cap [10000, +\infty)
\end{align*}\]

\[\begin{align*}
\text{←− no overflow}
\end{align*}\]

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\end{align*}\]

---

5 P. Cousot & R. Cousot, ISOP’1976, POPL’77.

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For imperative languages like PASCAL ...
An impressive application (1996/97)

A. Deutsch uses abstract interpretation (including interval analysis) for the static analysis of the embedded ADA software of the Ariane 5 launcher:

- Automatic detection of the definiteness, potentiality, impossibility or inaccessibility of run-time errors;
- Success for the 502 & 503 flights and the ARD.

Flight software (60,000 lines of Ada code) and Inertial Measurement Unit (30,000 lines of Ada code).

such as scalar and floating-point overflows, array index errors, divisions by zero and related arithmetic exceptions, uninitialized variables, data races on shared data structures, etc.

Atmospheric Reentry Demonstrator: module coming back to earth.
An impressive application (1996/97)

- A. Deutsch uses abstract interpretation (including interval analysis) for the static analysis of the embedded ADA software of the Ariane 5 launcher;

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• A. Deutsch uses abstract interpretation (including interval analysis) for the static analysis of the embedded ADA software of the Ariane 5 launcher \(^6\);
• Automatic detection of the definiteness, potentiality, impossibility or inaccessibility of run-time errors \(^7\);
• **Success** for the 502 & 503 flights and the ARD \(^8\).

---

\(^6\) Flight software (60,000 lines of Ada code) and Inertial Measurement Unit (30,000 lines of Ada code).

\(^7\) such as scalar and floating-point overflows, array index errors, divisions by zero and related arithmetic exceptions, uninitialized variables, data races on shared data structures, etc.

\(^8\) Atmospheric Reentry Demonstrator: module coming back to earth.
Some other recent applications of static analysis by abstract interpretation

- program transformation & optimization;
- abstract model-checking of infinite systems;
- abstract testing;
- type inference (for undecidable systems);
- mobile code communication topology;
- automatic differentiation;
- ...

Example of application of static analysis to program transformation & optimization
Example of application of static analysis to program transformation & optimization

The CLP(R) static analyser STAN

STAN input

The input parameters are:
1. The program:
   \[
   \begin{align*}
   &mc(A,B) : = A \geq 10, \ A - B = 10. \\
   &mc(A,B) : = A < 100, \ A - C = 11, \ mc(C,D), \ mc(D,B).
   \end{align*}
   \]

2. The goal:
   3. Type of the analysis: Backward

STAN output

\[
\begin{align*}
mc(A,B) [0] : &= A - B = 10, \ A \geq 101. \\
mc(A,B) [1] : &= E = 91, \ A \leq 100.
\end{align*}
\]
Some other recent applications of abstract interpretation

• Fundamental applications:
  – design of hierarchies of semantics,
  – …;

• Practical applications:
  – security (analysis of cryptographic protocols, mobile code),
  – semantic tattooing of software,
  – data mining,
  – ….

ongoing theses J. Feret, D. Monniaux
Lattice of semantics

Hoare logics

Weakest precondition semantics

Denotational semantics

Relational semantics

Trace semantics

angelic natural demoniac determinist infinite

abstraction equivalence restriction

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Forthcoming research

A lot of fundamental research remains to be one:

- modularity,
- higher order functions & modules,
- floating point numbers,
- probabilistic analyses,
- liveness properties with fairness,
- ...;
A few references

Starter:


On the web:

http://www.di.ens.fr/~cousot/
Industrialization of static analysis by abstract interpretation

- **First research results:** 1975;
- **First industrializations:**
  - [ ] Connected Components Corporation (U.S.A.), L. Harrison, 1993;
  - [ ] AbsInt Angewandte Informatik GmbH (Germany), R. Wilhelm, 1998;
Prospects

• The fundamental problems of computer science are difficult to explain to non specialists (only applications are well understood);
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• In the future, the society will certainly be better aware of these computer software related problems (e.g. through catastrophes);
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- The **fundamental problems** of computer science are difficult to explain to non-specialists (only applications are well understood);

- In the future, the society will certainly be better aware of these computer software related problems (e.g. through catastrophes);

- Research on **fundamental ideas** on software design is essential for modern societies;
Prospects

• The fundamental problems of computer science are difficult to explain to non specialists (only applications are well understood);

• In the future, the society will certainly be better aware of these computer software related problems (e.g. through catastrophes);

• Research on fundamental ideas on software design is essential for modern societies;

• The application of such fundamental research can hardly be scheduled in the short term (3 years);
Conclusion

Computer scientists need long term research funding.
Conclusion

Computer scientists need long term research funding.

THANK YOU FOR YOUR ATTENTION