SNU 4541.310 Programming Languages
(slides 3-1)

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의미구조(*semantics*)

- 프로그램이 뜻하는 바를 정의
- 프로그램이 뜻 하는 바?

"1+2" 라는 프로그램의 뜻?

- 의미 = 계산과정: "1과 2을 더해서 3을 계산한다."
  과정을 드러내는 의미구조(*operational semantics*)

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프로그램 실행 과정을 정의.

- 충분히 엄밀
- 조립식(compositional)이 아닐 수 있다
- 하지만 귀납적(inductive)이다
  - 프로그램 구조를 따라 되돌기도
  - 프로그램 이외의 것을 따라 되돌기도

리뷰: 집합의 정의, 추론 규칙

- 쌍 uncle(a, b)들의 집합

$$\frac{\text{male}(u) \quad \text{father}(f, i) \quad \text{brother}(f, u)}{\text{uncle}(u, i)}$$

$$\frac{\text{father}(c, a) \quad \text{father}(c, b)}{\text{brother}(a, b)}$$

$$\cdots$$

- 쌍 append(a, b, c)들의 집합

$$\frac{}{\text{append}(l, [], l)} \qquad \frac{}{\text{append}([], l, l)}$$

$$\frac{\text{append}(r, b, c)}{\text{append}(a :: r, b, a :: c)}$$

추론 과정 = 계산 (또는 “증명 나무”)

$$\dfrac{\dfrac{\dfrac{}{\mathsf{append}([], [3,4], [3,4])}}{\mathsf{append}([2], [3,4], [2,3,4])}}{\mathsf{append}([1,2], [3,4], [1,2,3,4])}$$

리뷰: 집합의 정의, 추론 규칙

쌍 $(\{g_1, \cdots, g_n\}, f)$들의 집합

$$\frac{}{(\Gamma, T)} \qquad \frac{}{(\Gamma, f)}\ f \in \Gamma \qquad\qquad \frac{(\Gamma, F)}{(\Gamma, f)} \qquad \frac{(\Gamma, \neg\neg f)}{(\Gamma, f)}$$

$$\frac{(\Gamma, f_1) \quad (\Gamma, f_2)}{(\Gamma, f_1 \wedge f_2)} \qquad\qquad \frac{(\Gamma, f_1 \wedge f_2)}{(\Gamma, f_1)}$$

$$\frac{(\Gamma, f_1)}{(\Gamma, f_1 \vee f_2)} \qquad\qquad \frac{(\Gamma, f_1 \vee f_2) \quad (\Gamma \cup \{f_1\}, f_3) \quad (\Gamma \cup \{f_2\}, f_3)}{(\Gamma, f_3)}$$

$$\frac{(\Gamma \cup \{f_1\}, f_2)}{(\Gamma, f_1 \Rightarrow f_2)} \qquad\qquad \frac{(\Gamma, f_1 \Rightarrow f_2) \quad (\Gamma, f_1)}{(\Gamma, f_2)}$$

$$\frac{(\Gamma \cup \{f\}, F)}{(\Gamma, \neg f)} \qquad\qquad \frac{(\Gamma, f) \quad (\Gamma, \neg f)}{(\Gamma, F)}$$

형식논리의 표기법으로

$$\frac{}{\Gamma \vdash T} \quad \frac{}{\Gamma \vdash f}\ f \in \Gamma \qquad\qquad \frac{\Gamma \vdash F}{\Gamma \vdash f} \quad \frac{\Gamma \vdash \neg\neg f}{\Gamma \vdash f}$$

$$\frac{\Gamma \vdash f_1 \quad \Gamma \vdash f_2}{\Gamma \vdash f_1 \wedge f_2} \qquad\qquad \frac{\Gamma \vdash f_1 \wedge f_2}{\Gamma \vdash f_1}$$

$$\frac{\Gamma \vdash f_1}{\Gamma \vdash f_1 \vee f_2} \qquad\qquad \frac{\Gamma \vdash f_1 \vee f_2 \quad \Gamma \cup \{f_1\} \vdash f_3 \quad \Gamma \cup \{f_2\} \vdash f_3}{\Gamma \vdash f_3}$$

$$\frac{\Gamma \cup \{f_1\} \vdash f_2}{\Gamma \vdash f_1 \Rightarrow f_2} \qquad\qquad \frac{\Gamma \vdash f_1 \Rightarrow f_2 \quad \Gamma \vdash f_1}{\Gamma \vdash f_2}$$

$$\frac{\Gamma \cup \{f\} \vdash F}{\Gamma \vdash \neg f} \qquad\qquad \frac{\Gamma \vdash f \quad \Gamma \vdash \neg f}{\Gamma \vdash F}$$

추론 과정 = 계산 (또는 "증명 나무")

$$\dfrac{\dfrac{\dfrac{}{\{p \to \neg p, p\} \vdash p} \qquad \dfrac{\dfrac{}{\{p \to \neg p, p\} \vdash p \to \neg p} \qquad \dfrac{}{\{p \to \neg p, p\} \vdash p}}{\{p \to \neg p, p\} \vdash \neg p}}{\{p \to \neg p, p\} \vdash F}}{\{p \to \neg p\} \vdash \neg p}$$

의미공간(*semantic domain*)

보통의 집합

- 귀납법, 원소나열법, 조건제시법, $S \cup T, S + T, S \times T, S \xrightarrow{\text{fin}} T$

$$S \xrightarrow{\text{fin}} T = \{f | f \in S' \to T, S' \overset{\text{fin}}{\subseteq} S\}$$

- 일반 함수집합 만들기 $\to$를 사용하면 곤란

프로그램의 의미 = 증명

structural operational semantics,

natural semantics, relational semantics

- 증명 규칙(*inference rule*): 프로그램 구조마다 하나 이상 씩

$$\frac{\cdots}{(M, \texttt{skip}, M')} \qquad \frac{\cdots}{(M, x \texttt{ := } E, M')}$$

$$\frac{\cdots}{(M, C_1 \texttt{ ; } C_2, M')} \qquad \frac{\cdots}{(M, \texttt{if } E\ C_1\ C_2, M')}$$

$$\frac{\cdots}{(M, \texttt{while } E\ C, M')}$$

- 무한한 실행과정? 무한한 증명

예

명령문 C와 정수식 e의 의미는 (메모리 M 에서)

$$M \vdash C \Rightarrow M' \quad \text{와} \quad M \vdash e \Rightarrow v$$

의 증명

- 증명 불가능 $\Leftrightarrow$ C는 메모리 M에서 무의미

큰 보폭으로(*big-step operational semantics*)

$$\begin{array}{rcrcl} M & \in & \mathit{Memory} & = & \mathit{Var} \xrightarrow{\text{fin}} \mathit{Val} \\ v & \in & \mathit{Val} & = & \mathbb{Z} \end{array}$$

$$\frac{}{M \vdash \texttt{skip} \Rightarrow M}$$

$$\frac{M \vdash E \Rightarrow v}{M \vdash x \texttt{ := } E \Rightarrow M\{x \mapsto v\}}$$

$$\frac{M \vdash C_1 \Rightarrow M_1 \quad M_1 \vdash C_2 \Rightarrow M_2}{M \vdash C_1 \texttt{ ; } C_2 \Rightarrow M_2}$$

$$\frac{M \vdash E \Rightarrow 0 \quad M \vdash C_2 \Rightarrow M'}{M \vdash \texttt{if}\ E\ C_1\ C_2 \Rightarrow M'}$$

$$\frac{M \vdash E \Rightarrow v \quad M \vdash C_1 \Rightarrow M'}{M \vdash \texttt{if}\ E\ C_1\ C_2 \Rightarrow M'}\ v \neq 0$$

$$\frac{M \vdash E \Rightarrow 0}{M \vdash \texttt{while}\ E\ C \Rightarrow M}$$

$$\frac{M \vdash E \Rightarrow v \quad M \vdash C \Rightarrow M_1 \quad M_1 \vdash \texttt{while}\ E\ C \Rightarrow M_2}{M \vdash \texttt{while}\ E\ C \Rightarrow M_2}\ v \neq 0$$

$$\frac{}{M \vdash n \Rightarrow n}$$

$$\frac{}{M \vdash x \Rightarrow M(x)}$$

$$\frac{M \vdash E_1 \Rightarrow v_1 \quad M \vdash E_2 \Rightarrow v_2}{M \vdash E_1 \texttt{+} E_2 \Rightarrow v_1 + v_2}$$

$$\frac{M \vdash E \Rightarrow v}{M \vdash \texttt{-} E \Rightarrow -v}$$

$$C \stackrel{\text{let}}{=} x \mathrel{\mathtt{:=}} \mathtt{1} \; ; \; y \mathrel{\mathtt{:=}} x \mathbin{\mathtt{+}} \mathtt{1}$$

$$\dfrac{\dfrac{\emptyset \vdash \mathtt{1} \Rightarrow 1}{\emptyset \vdash x \mathrel{\mathtt{:=}} \mathtt{1} \Rightarrow \{x \mapsto 1\}} \qquad \dfrac{\dfrac{\{x \mapsto 1\} \vdash x \Rightarrow 1 \qquad \{x \mapsto 1\} \vdash \mathtt{1} \Rightarrow 1}{\{x \mapsto 1\} \vdash x \mathbin{\mathtt{+}} \mathtt{1} \Rightarrow 2}}{\{x \mapsto 1\} \vdash y \mathrel{\mathtt{:=}} x \mathbin{\mathtt{+}} \mathtt{1} \Rightarrow \{x \mapsto 1, y \mapsto 2\}}}{\emptyset \vdash C \Rightarrow \{x \mapsto 1, y \mapsto 2\}}$$