

Engineering Mathematics II - HW 1 Solution

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Exercise 2 $\forall x \in \text{Exp.}$ if every variable in e has an even integer then e 's value is an even integer. We will prove it by induction on e .

- **Base case** $0 \mid 2$: even integers
- **Base case** x : even integer ($\because$ every variable in e has an even integer)
- **Inductive case** $e_1 - e_2 \mid e_1 \times e_2$: by I.H on e_1, e_2 the values of e_1 and e_2 are even integers. Both difference and multiplication of even integers are also even.

□

Exercise 3 proof construction

- $\neg(p \vee q) \vdash \neg p \wedge \neg q$

style 1
 $\neg(p \vee q)$

$$\frac{\left(\frac{\frac{\neg(\neg p \wedge \neg q)}{\left(\frac{\frac{p}{p \vee q}}{0} \right)} \quad \frac{\frac{q}{p \vee q}}{0} \right)}{\neg p \quad \neg q} \quad \frac{\neg p \wedge \neg q}{0} \quad \frac{\neg \neg(\neg p \wedge \neg q)}{\neg p \wedge \neg q}$$

style 2
let $G = \neg(p \vee q)$

$$\frac{\frac{\frac{\overline{G, p \vdash p}}{G, p \vdash p \vee q} \quad \frac{\overline{G, p \vdash \neg(p \vee q)}}{G, p \vdash 0}}{G \vdash \neg p} \quad \frac{\frac{\frac{\overline{G, q \vdash q}}{G, q \vdash p \vee q} \quad \frac{\overline{G, q \vdash \neg(p \vee q)}}{G, q \vdash 0}}{G \vdash \neg q}}{G \vdash (\neg p \wedge \neg q)}$$

- $\neg(p \wedge q) \vdash \neg p \vee \neg q$

style 1

$\neg(p \wedge q)$

$$\frac{\left(\frac{\frac{\neg(\neg p \vee \neg q)}{\left(\frac{\frac{\neg p}{\neg p \vee \neg q}}{0} \right)} \quad \frac{\frac{\neg q}{\neg p \vee \neg q}}{0} \right)}{\frac{\neg\neg p}{p} \quad \frac{\neg\neg q}{q}} \frac{p \wedge q}{0} \frac{\neg\neg(\neg p \vee \neg q)}{\neg p \vee \neg q}$$

style 2

let $G = \neg(p \wedge q), \neg F$

and $F = (\neg p \vee \neg q)$

$$\frac{\frac{\frac{\overline{G, \neg p \vdash \neg p}}{G, \neg p \vdash F} \quad \frac{\overline{G, \neg p \vdash \neg F}}{G, \neg p \vdash 0}}{G \vdash \neg\neg p} \quad \frac{\frac{\frac{\overline{G, \neg q \vdash \neg q}}{G, \neg q \vdash F} \quad \frac{\overline{G, \neg q \vdash \neg F}}{G, \neg q \vdash 0}}{G \vdash \neg\neg q}}{G \vdash q} \frac{G \vdash p \wedge q}{G \vdash 0} \frac{G \vdash \neg(p \wedge q)}{\neg(p \wedge q) \vdash \neg\neg F} \neg(p \wedge q) \vdash F$$

- $p \vee (q \wedge r) \vdash (p \vee q) \wedge (p \vee r)$

style 1

$$\frac{\left(\frac{\frac{p}{p \vee q} \quad \frac{p}{p \vee r}}{(p \vee q) \wedge (p \vee r)} \right) \quad p \vee (q \wedge r) \quad \left(\frac{\frac{\frac{q \wedge r}{q} \quad \frac{q \wedge r}{r}}{p \vee q \quad p \vee r}}{(p \vee q) \wedge (p \vee r)} \right)}{(p \vee q) \wedge (p \vee r)}$$

style 2

let $G = p \vee (q \wedge r)$

and $F = (p \vee q) \wedge (p \vee r)$

$$\frac{\frac{\frac{\overline{G, p \vdash p}}{G, p \vdash p \vee q} \quad \frac{\overline{G, p \vdash p}}{G, p \vdash p \vee r}}{G, p \vdash F} \quad \frac{\overline{G \vdash p \vee (q \wedge r)}}{G \vdash F} \quad \frac{\frac{\frac{\overline{G, q \wedge r \vdash q \wedge r}}{G, q \wedge r \vdash q} \quad \frac{\overline{G, q \wedge r \vdash q \wedge r}}{G, q \wedge r \vdash r}}{G, q \wedge r \vdash p \vee q} \quad \frac{\overline{G, q \wedge r \vdash q \wedge r}}{G, q \wedge r \vdash p \vee r}}{G, q \wedge r \vdash F}$$

- $p \wedge q, (p \wedge q \rightarrow r) \vdash (p \rightarrow r) \wedge (q \rightarrow r)$

style 1

$$p \wedge q, (p \wedge q \rightarrow r)$$

$$\frac{\frac{\frac{\frac{q}{p \rightarrow r}}{p \wedge q}}{q \rightarrow (p \rightarrow r)} \quad \frac{\frac{\frac{\frac{p}{q \rightarrow r}}{p \wedge q}}{p \rightarrow (q \rightarrow r)}}{(p \rightarrow r) \wedge (q \rightarrow r)}$$

style 2

$$\text{let } G = p \wedge q, (p \wedge q \rightarrow r)$$

$$\frac{\frac{\frac{G, p \vdash p \wedge q}{G, p \vdash r}}{G \vdash p \rightarrow r} \quad \frac{\frac{\frac{G, q \vdash p \wedge q}{G, q \vdash r}}{G \vdash q \rightarrow r}}{G \vdash (p \rightarrow r) \wedge (q \rightarrow r)}$$

□

Exercise 4 *proof = program*

- Construct the proof tree

$$\text{let } G = (p \wedge q) \wedge (p \wedge q \rightarrow r)$$

$$\frac{\frac{\frac{G, p \vdash p \wedge q \rightarrow r}{(p \wedge q) \wedge (p \wedge q \rightarrow r), p \vdash r} \quad \frac{\frac{G, q \vdash p \wedge q \rightarrow r}{(p \wedge q) \wedge (p \wedge q \rightarrow r), q \vdash r}}{(p \wedge q) \wedge (p \wedge q \rightarrow r) \vdash (p \rightarrow r) \wedge (q \rightarrow r)}}{(p \wedge q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r) \wedge (q \rightarrow r)}$$

- Construct a program

$$\lambda f. (\lambda x. (f.2 \ f.1), \lambda y. (f.2 \ f.1))$$

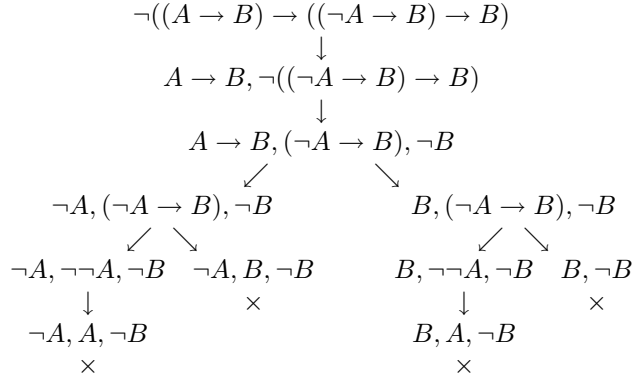
Exercise 5 *Ex 3.7*

- 1. Prove in G : $\vdash A$ in G iff there is a closed semantic tableau for $\neg A$.

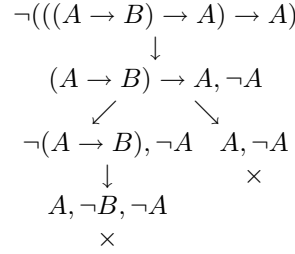
$$\vdash (A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

$$\begin{array}{c} \neg((A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)) \\ \downarrow \\ A \rightarrow B, \neg(\neg B \rightarrow \neg A) \\ \downarrow \\ A \rightarrow B, \neg B, \neg \neg A \\ \downarrow \\ A \rightarrow B, \neg B, A \\ \swarrow \quad \searrow \\ \neg A, \neg B, A \quad B, \neg B, A \\ \times \quad \times \end{array}$$

$$\vdash (A \rightarrow B) \rightarrow ((\neg A \rightarrow B) \rightarrow B)$$

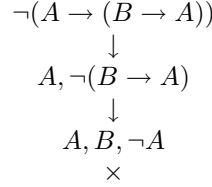


$$\vdash ((A \rightarrow B) \rightarrow A) \rightarrow A$$

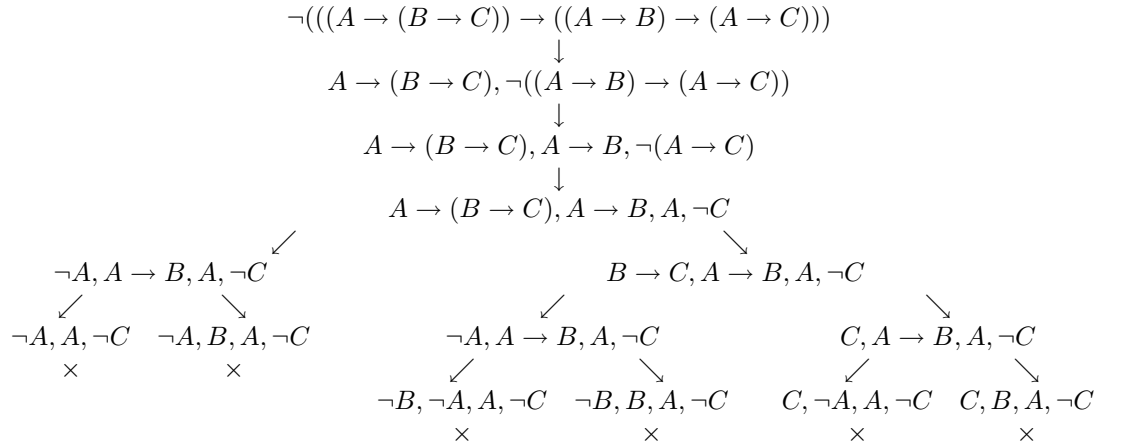


- 4. Give proofs in G for each of the three axioms of H .

$$\vdash (A \rightarrow (B \rightarrow A)).$$



$$\vdash ((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))).$$



$$\vdash (\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B).$$

$$\begin{array}{c}
\neg((\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)) \\
\downarrow \\
(\neg B \rightarrow \neg A), \neg(A \rightarrow B) \\
\downarrow \\
(\neg B \rightarrow \neg A), A, \neg B \\
\swarrow \quad \searrow \\
B, A, \neg B \quad \neg A, A, \neg B \\
\times \qquad \times
\end{array}$$