

Engineering Mathematics II - Quiz 2 Solution

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1. (20pts) Resolution으로 옳다는 것을 증명하겠다.

$\{\neg(\text{Home}(\text{Max}) \wedge \text{Home}(\text{Claire})), \text{Home}(\text{Max}) \vee \text{Happy}(\text{Carl}), \text{Home}(\text{Claire}) \vee \text{Happy}(\text{Carl}), \neg\text{Happy}(\text{Carl})\}$
 $\{\neg\text{Home}(\text{Max}) \vee \neg\text{Home}(\text{Claire}), \text{Home}(\text{Max}) \vee \text{Happy}(\text{Carl}), \text{Home}(\text{Claire}) \vee \text{Happy}(\text{Carl}), \neg\text{Happy}(\text{Carl})\}$
 $\{\neg\text{Home}(\text{Claire}) \vee \text{Happy}(\text{Carl}), \text{Home}(\text{Claire}) \vee \text{Happy}(\text{Carl}), \neg\text{Happy}(\text{Carl})\}$
 $\{\text{Happy}(\text{Carl}), \neg\text{Happy}(\text{Carl})\}$
 $\{;;\}$

□

2. (20pts) BackOf(a,b)는 참이다. Resolution으로 옳다는 것을 증명하겠다. 우선 문제의 식에서 FrontOf(b,a)는 BackOf(a,b)로 의미가 같으므로 바꿀 수 있다.

$\{\text{LeftOf}(a,b) \vee \text{RightOf}(a,b), \text{BackOf}(a,b) \vee \neg\text{LeftOf}(a,b),$
 $\text{BackOf}(a,b) \vee \neg\text{RightOf}(a,b), \text{Large}(c), \neg\text{BackOf}(a,b)\}$

 $\{\text{RightOf}(a,b) \vee \text{BackOf}(a,b), \text{BackOf}(a,b) \vee \neg\text{RightOf}(a,b), \text{Large}(c), \neg\text{BackOf}(a,b)\}$
 $\{\text{BackOf}(a,b) \vee \text{BackOf}(a,b), \text{Large}(c), \neg\text{BackOf}(a,b)\}$
 $\{\text{BackOf}(a,b), \text{Large}(c), \neg\text{BackOf}(a,b)\}$
 $\{;;; \text{Large}(c)\}$

□

3. (20pts) 강의자료 8번의 첫 페이지 참조.

4. (20pts) Prove, by using the proof rules, the equivalence: $\neg(\forall x.p) \sim \exists x.(\neg p)$.
 $\neg(\forall x.p) \vdash \exists x.(\neg p)$ 이고 $\exists x.(\neg p) \vdash \neg(\forall x.p)$ 이면 $\neg(\forall x.p) \sim \exists x.(\neg p)$ 이다.

- $\neg(\forall x.p) \vdash \exists x.(\neg p)$

$$\neg(\forall x.p) \vdash \left(\frac{\left(\frac{\left(\frac{\neg\exists x.(\neg p)}{\neg p} \right)}{\exists x.(\neg p)} \right)}{0} \right)}{\neg\neg p} \frac{p}{\forall x.p} \frac{0}{\neg\neg\exists x.(\neg p)} \frac{\exists x.(\neg p)}{\exists x.(\neg p)}$$

- $\exists x.(\neg p) \vdash \neg(\forall x.p)$

$$\exists x.(\neg p) \quad \left(\frac{\frac{\frac{\neg\neg\forall x.p}{\forall x.p}}{\neg\{\exists x.(\neg p)\}}}{0} \right) \frac{}{\frac{\neg\neg\neg(\forall x.p)}{\neg(\forall x.p)}}$$

5. (20pts) Find a model that the following formula is true:

$$\forall x.\exists y.(\kappa(f(x), g(y)) \wedge \neg\phi(h(x, y))).$$

- 임의의 모델 $\mathcal{M} = \{\mathcal{D}, \mathcal{I}\}$ 중 하나의 예를 들면,

<i>Domain</i>	$x \in \mathcal{N}, y \in \mathcal{N}$
	$f(x) = 10$
	$g(x) = 10$
	$h(x, y) = 1$
<i>Interpretation</i>	$\kappa(x, y) = \begin{cases} \text{true,} & \text{when } x = y \\ \text{false,} & \text{otherwise} \end{cases}$
	$\phi(x) = \begin{cases} \text{false,} & \text{when } x = 1 \\ \text{true,} & \text{otherwise} \end{cases}$

$$\mathcal{M} \models \forall x.\exists y.(\kappa(f(x), g(y)) \wedge \neg\phi(h(x, y))).$$