4190.310 Programming Language
Semeantics of K--

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$$\text{TRUE}\ \frac{}{\sigma, M \vdash \textbf{true} \Rightarrow true, M} \qquad \text{FALSE}\ \frac{}{\sigma, M \vdash \textbf{false} \Rightarrow false, M}$$

$$\text{NUM}\ \frac{}{\sigma, M \vdash \textbf{n} \Rightarrow n, M} \qquad \text{UNIT}\ \frac{}{\sigma, M \vdash \textbf{unit} \Rightarrow \cdot, M}$$

$$\text{VAR}\ \frac{}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

$$\text{ADD}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 + E_2 \Rightarrow n_1 + n_2, M''}$$

$$\text{SUB}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 \text{-} E_2 \Rightarrow n_1 - n_2, M''}$$

$$\text{MUL}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 \times E_2 \Rightarrow n_1 \times n_2, M''}$$

$$\text{DIV}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 \div E_2 \Rightarrow n_1 \div n_2, M''}$$

$$\text{EQUAL}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 = E_2 \Rightarrow n_1 = n_2, M''}$$

$$\text{LESS}\ \frac{\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M''}{\sigma, M \vdash E_1 < E_2 \Rightarrow n_1 < n_2, M''}$$

$$\text{NOT}\ \frac{\sigma, M \vdash E \Rightarrow b, M'}{\sigma, M \vdash \text{not} E \Rightarrow not\, b, M'}$$

$$\text{ASSIGN}\ \frac{\sigma, M \vdash E \Rightarrow v, M'}{\sigma, M \vdash x := E \Rightarrow v, M'\{\sigma(x) \mapsto v\}}$$

$$\text{SEQ}\ \frac{\sigma, M \vdash E_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow v_2, M''}{\sigma, M \vdash E_1; E_2 \Rightarrow v_2, M''}$$

$$\text{IFT}\ \frac{\sigma, M \vdash E \Rightarrow true, M' \qquad \sigma, M' \vdash E_1 \Rightarrow v, M''}{\sigma, M \vdash \text{if } E \text{ then } E_1 \text{ else } E_2 \Rightarrow v, M''}$$

$$\text{IFF}\ \frac{\sigma, M \vdash E \Rightarrow false, M' \qquad \sigma, M' \vdash E_2 \Rightarrow v, M''}{\sigma, M \vdash \text{if } E \text{ then } E_1 \text{ else } E_2 \Rightarrow v, M''}$$

$$\text{WHILEF}\ \frac{\sigma, M \vdash E_1 \Rightarrow false, M'}{\sigma, M \vdash \text{while } E_1 \text{ do } E_2 \Rightarrow \cdot, M'}$$

$$\text{WHILET}\ \frac{\begin{array}{c}\sigma, M \vdash E_1 \Rightarrow true, M' \\ \sigma, M' \vdash E_2 \Rightarrow v_1, M_1 \qquad \sigma, M_1 \vdash \text{while } E_1 \text{ do } E_2 \Rightarrow v_2, M_2\end{array}}{\sigma, M \vdash \text{while } E_1 \text{ do } E_2 \Rightarrow v_2, M'}$$

$$\text{FOR}\ \frac{\begin{array}{c}\sigma, M \vdash E_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash E_2 \Rightarrow n_2, M'' \\ \sigma, M''\{\sigma(x) \mapsto n_1 + 0\} \vdash C \Rightarrow v_0, M_0 \\ \vdots \\ \sigma, M_{n_2-n_1-1}\{\sigma(x) \mapsto n_1 + (n_2 - n_1)\} \vdash C \Rightarrow v_{n_2-n_1}, M_{n_2-n_1}\end{array}}{\sigma, M \vdash \text{for } x := E_1 \text{ to } E_2 \text{ do } C \Rightarrow \cdot, M_{n_2-n_1}}\ n_2 \geq n_1$$

$$\text{LETV}\ \frac{\begin{array}{c}\sigma, M \vdash E_1 \Rightarrow v, M' \\ \sigma\{x \mapsto l\}, M'\{l \mapsto v\} \vdash E_2 \Rightarrow v', M''\end{array}}{\sigma, M \vdash \text{let } x := E_1 \text{ in } E_2 \Rightarrow v', M''}\ l \notin Dom\, M'$$

$$\text{LETF}\ \frac{\sigma\{f \mapsto < x, E_1, \sigma >\}, M \vdash E_2 \Rightarrow v, M'}{\sigma, M \vdash \text{let proc } f(x) \text{ in } E_2 \Rightarrow v, M'}$$

$$\text{CALLV}\ \frac{\begin{array}{c}\sigma, M \vdash E \Rightarrow v, M' \\ \sigma'\{x \mapsto l\}\{f \mapsto < x, E', \sigma' >\}, M'\{l \mapsto v\} \vdash E' \Rightarrow v', M''\end{array}}{\sigma, M \vdash f(E) \Rightarrow v', M''}\ \begin{array}{l}\sigma(f) = < x, E', \sigma' > \\ l \notin Dom\, M'\end{array}$$

$$\text{CALLR}\ \frac{\sigma'\{x \mapsto \sigma(y)\}\{f \mapsto < x, E, \sigma' >\}, M \vdash E \Rightarrow M'}{\sigma, M \vdash f < y > \Rightarrow M'}\ \sigma(f) = < x, E, \sigma' >$$

$$\text{READ}\ \frac{}{\sigma, M \vdash \text{read } x \Rightarrow n, M\{\sigma(x) \mapsto n\}}$$

$$\text{WRITE}\ \frac{\sigma, M \vdash E \Rightarrow v, M'}{\sigma, M \vdash \text{write } E \Rightarrow v, M'}$$