

4190.310 Programming Language

The K- Language

2011 Spring

1 Syntax

<i>Expression e</i>	→	unit	unit
		<i>x := e</i>	assignment
		<i>e ; e</i>	sequence
		if <i>e</i> then <i>e</i> else <i>e</i>	branch
		while <i>e</i> do <i>e</i>	while loop
		read <i>x</i>	input
		write <i>e</i>	output
		let <i>x := e</i> in <i>e</i>	variable binding
		let proc <i>f(x₁, x₂, ..., x_n) = e</i> in <i>e</i>	procedure binding
		<i>f(e₁, e₂, ..., e_n)</i>	call by value
		<i>f</i> < <i>x₁, x₂, ..., x_n</i> >	call by reference
		<i>n</i>	integer
		true false	boolean
		{} {x₁ := e₁, x₂ := e₂, ..., x_n := e_n}	record
		<i>e.x</i>	record lookup
		<i>e.x := e</i>	record assignment
		<i>x</i>	identifier
		<i>e + e</i> <i>e - e</i> <i>e * e</i> <i>e / e</i>	arithmetic operation
		<i>e < e</i> <i>e = e</i> not <i>e</i>	conditional operation
		malloc <i>e</i> &x &e.x	
		*e *e := e	locations as values

1.1 Program

A program is an expression.

1.2 Identifiers

Alpha-numeric identifiers are $[a-zA-Z][a-zA-Z0-9_]^*$. Identifiers are case sensitive: `z` and `Z` are different. The reserved words cannot be used as identifiers: `unit true false not if then else let in proc while do read write malloc`.

1.3 Numbers/Comments

Numbers are integers, optionally prefixed with `-` (for negative integer): $-?[0-9]^+$.

A comment is any character sequence within the comment block `(* *)`. Comment blocks can be nested.

1.4 Precedence/Associativity

In parsing K- program text, the precedence of K- constructs in decreasing order is as follows. Symbols in the same set have identical precedence. Symbols with subscript *L* (respectively *R*) are left (respectively right) associative. Symbols without subscript are nonassociative.

$\{.\}_L$,
 $\{\text{not, malloc, \&, *}\}_R$,
 $\{*, /\}_L$,
 $\{+, -\}_L$,
 $\{=, <\}_L$,
 $\{\text{write}\}_R$,
 $\{:=\}_R$,
 $\{\text{else}\}$,
 $\{\text{then}\}$,
 $\{\text{do}\}$,
 $\{;\}_L$,
 $\{\text{in}\}$

For example, K- program

$$\begin{aligned}x := e1; e2 &\Rightarrow (x := e1) ; e2 \\ \text{while } e \text{ do } e1; e2 &\Rightarrow (\text{while } e \text{ do } e1); e2 \\ \text{if } e1 \text{ then } e2 \text{ else } e3; e4 &\Rightarrow (\text{if } e1 \text{ then } e2 \text{ else } e3); e4\end{aligned}$$

Rule of thumb: If your test programs are hard to read (hence can be parsed not as you expected) then put parentheses around.

2 Domains

$$\begin{aligned}n &\in \mathbb{Z} && \text{integer} \\ b &\in \mathbb{B} && \text{boolean} \\ r &\in \text{Record} = Id \xrightarrow{fin} Addr \\ v &\in \text{Val} = \mathbb{Z} + \mathbb{B} + \{\cdot\} + \text{Record} + Addr \\ \sigma &\in \text{Env} = Id \xrightarrow{fin} Addr + Procedure \\ M &\in \text{Mem} = Addr \xrightarrow{fin} Val \\ x, y &\in Id && \text{identifier} \\ l &\in Addr && \text{address} \\ Procedure &= (Id \times Id \times \dots) \times Expression \times Env\end{aligned}$$

3 Semantics

$$\begin{array}{c}
\text{TRUE} \frac{}{\sigma, M \vdash \mathbf{true} \Rightarrow \mathit{true}, M} \qquad \text{FALSE} \frac{}{\sigma, M \vdash \mathbf{false} \Rightarrow \mathit{false}, M} \\
\\
\text{NUM} \frac{}{\sigma, M \vdash \mathbf{n} \Rightarrow n, M} \qquad \text{UNIT} \frac{}{\sigma, M \vdash \mathbf{unit} \Rightarrow \cdot, M} \\
\\
\text{VAR} \frac{}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M} \\
\\
\text{RECF} \frac{}{\sigma, M \vdash \{\} \Rightarrow \cdot, M} \\
\\
\begin{array}{c}
\sigma, M \vdash e_1 \Rightarrow v_1, M_1 \\
\sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \\
\vdots \\
\sigma, M_{n-1} \vdash e_n \Rightarrow v_n, M_n
\end{array} \\
\text{RECT} \frac{}{\sigma, M \vdash \{x_1 := e_1, \dots, x_n := e_n\} \Rightarrow \{x_1 \mapsto l_1, \dots, x_n \mapsto l_n\}, M_n \{l_1 \mapsto v_1, \dots, l_n \mapsto v_n\}} \quad l_i \notin \text{Dom } M_n \\
\\
\text{ADD} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 + e_2 \Rightarrow n_1 + n_2, M''} \\
\\
\text{SUB} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 - e_2 \Rightarrow n_1 - n_2, M''} \\
\\
\text{MUL} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 * e_2 \Rightarrow n_1 * n_2, M''} \\
\\
\text{DIV} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 / e_2 \Rightarrow n_1/n_2, M''}
\end{array}$$

$$\text{EQUALT} \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow v_2, M'' \quad \begin{array}{l} v_1 = v_2 = n \\ \vee v_1 = v_2 = b \\ \vee v_1 = v_2 = . \end{array}}{\sigma, M \vdash e_1 = e_2 \Rightarrow \mathbf{true}, M''}$$

$$\text{EQUALF} \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 = e_2 \Rightarrow \mathbf{false}, M''} \text{ otherwise}$$

$$\text{LESS} \frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 < e_2 \Rightarrow n_1 < n_2, M''}$$

$$\text{NOT} \frac{\sigma, M \vdash e \Rightarrow b, M'}{\sigma, M \vdash \mathbf{not} \ e \Rightarrow \mathbf{not} \ b, M'}$$

$$\text{ASSIGN} \frac{\sigma, M \vdash e \Rightarrow v, M'}{\sigma, M \vdash x := e \Rightarrow v, M' \{ \sigma(x) \mapsto v \}}$$

$$\text{RECASSIGN} \frac{\sigma, M \vdash e_1 \Rightarrow r, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v, M_2}{\sigma, M \vdash e_1 . x := e_2 \Rightarrow v, M_2 \{ r(x) \mapsto v \}}$$

$$\text{RECLOOKUP} \frac{\sigma, M \vdash e \Rightarrow r, M'}{\sigma, M \vdash e . x \Rightarrow M'(r(x)), M'}$$

$$\text{SEQ} \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \quad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 ; e_2 \Rightarrow v_2, M''}$$

$$\text{IFT} \frac{\sigma, M \vdash e \Rightarrow \mathbf{true}, M' \quad \sigma, M' \vdash e_1 \Rightarrow v, M''}{\sigma, M \vdash \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2 \Rightarrow v, M''}$$

$$\text{IFF} \frac{\sigma, M \vdash e \Rightarrow \mathbf{false}, M' \quad \sigma, M' \vdash e_2 \Rightarrow v, M''}{\sigma, M \vdash \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2 \Rightarrow v, M''}$$

$$\begin{array}{c}
\text{WHILEF} \frac{\sigma, M \vdash e_1 \Rightarrow \text{false}, M'}{\sigma, M \vdash \text{while } e_1 \text{ do } e_2 \Rightarrow \cdot, M'} \\
\\
\text{WHILET} \frac{\sigma, M \vdash e_1 \Rightarrow \text{true}, M' \quad \sigma, M_1 \vdash \text{while } e_1 \text{ do } e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash \text{while } e_1 \text{ do } e_2 \Rightarrow v_2, M_2} \\
\\
\text{LETV} \frac{\sigma, M \vdash e_1 \Rightarrow v, M' \quad \sigma\{x \mapsto l\}, M'\{l \mapsto v\} \vdash e_2 \Rightarrow v', M''}{\sigma, M \vdash \text{let } x := e_1 \text{ in } e_2 \Rightarrow v', M''} \quad l \notin \text{Dom } M' \\
\\
\text{LETF} \frac{\sigma\{f \mapsto \langle (x_1, \dots, x_n), e_1, \sigma \rangle\}, M \vdash e_2 \Rightarrow v, M'}{\sigma, M \vdash \text{let proc } f(x_1, \dots, x_n) = e_1 \text{ in } e_2 \Rightarrow v, M'} \\
\\
\text{CALLV} \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \quad \vdots \quad \sigma, M_{n-1} \vdash e_n \Rightarrow v_n, M_n \quad \sigma'\{x_1 \mapsto l_1\} \cdots \{x_n \mapsto l_n\} \{f \mapsto \langle (x_1, \dots, x_n), e', \sigma' \rangle\}, \quad M_n\{l_1 \mapsto v_1\} \cdots \{l_n \mapsto v_n\} \vdash e' \Rightarrow v', M'}{\sigma, M \vdash f(e_1, \dots, e_n) \Rightarrow v', M'} \quad \begin{array}{l} \sigma(f) = \langle (x_1, \dots, x_n), e', \sigma' \rangle \\ l_i \notin \text{Dom } M' \end{array} \\
\\
\text{CALLR} \frac{\sigma'\{x_1 \mapsto \sigma(y_1)\} \cdots \{x_n \mapsto \sigma(y_n)\} \{f \mapsto \langle (x_1, \dots, x_n), e, \sigma' \rangle\}, \quad M \vdash e \Rightarrow v, M'}{\sigma, M \vdash f\langle y_1, \dots, y_n \rangle \Rightarrow v, M'} \quad \sigma(f) = \langle (x_1, \dots, x_n), e, \sigma' \rangle \\
\\
\text{READ} \frac{}{\sigma, M \vdash \text{read } x \Rightarrow n, M\{\sigma(x) \mapsto n\}} \\
\\
\text{WRITE} \frac{\sigma, M \vdash e \Rightarrow v, M'}{\sigma, M \vdash \text{write } e \Rightarrow v, M'}
\end{array}$$

$$\text{MALLOC} \frac{\sigma, M \vdash e \Rightarrow n, M'}{\sigma, M \vdash \text{malloc } e \Rightarrow l, M'} \quad n > 0, \{l, l+1, \dots, l+n-1\} \not\subseteq \text{dom}(M')$$

$$\text{AMPER} \frac{}{\sigma, M \vdash \&x \Rightarrow \sigma(x), M}$$

$$\text{AMPERFIELD} \frac{\sigma, M \vdash e \Rightarrow r, M'}{\sigma, M \vdash \&e.x \Rightarrow r(x), M'}$$

$$\text{STAR} \frac{\sigma, M \vdash e \Rightarrow l, M'}{\sigma, M \vdash *e \Rightarrow M'(l), M'}$$

$$\text{ASSIGNSTAR} \frac{\sigma, M \vdash e_1 \Rightarrow l, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v, M_2}{\sigma, M \vdash *e_1 := e_2 \Rightarrow v, M_2\{l \mapsto v\}}$$