Homework 3 SNU 4541.664A, 2010 Spring

Woo suk Lee

2010년 4월 6일

Exercise 1 *P($fixF_w$, $fixF_r$) ≜ (∀ C,E.($fixF_w$) ∘ {|C|} = $fixF_r$)*

Proof.
P는 품에 넣는 성질(inclusive assertion)을 만족시키므로, 고정점 귀납법으로 P(⊥, ⊥) 이고, P(f,g) ⇒ P($F_w f$, $F_r g$) 임을 보이면 된다.

$$F_w = \lambda X.\lambda M.if\ \{\!|E|\!\}M \neq 0\ then\ X(\{\!|C|\!\}M)\ else\ M$$
$$F_r = \lambda X.(\lambda M.if\ \{\!|E|\!\}M\ then\ X(M)\ else\ M \circ \{\!|C|\!\})$$

1) P(⊥, ⊥)

$$P(\perp, \perp) = \forall C, E\ (\perp_{Mem\rightarrow Mem}) \circ \{\!|C|\!\} = \perp_{Mem\rightarrow Mem}$$

2) P(f,g) ⇒ P($F_w f$, $F_r g$)

$$
\begin{aligned}
P(f, g) &= \forall C, E\ f \circ \{\!|C|\!\} = g \\
F_w f &= (\lambda M.if\{\!|E|\!\}M \neq 0\ \ then\ \ f(\{\!|C|\!\}M)\ else\ M) \\
&= (\lambda M.if\{\!|E|\!\}M \neq 0\ then\ g(M)\ else\ M) \\
&\quad (\because \forall C, E\ f \circ \{\!|C|\!\} = g) \\
F_r g &= (\lambda M.if\{\!|E|\!\}M\ then\ g(M)\ else\ M) \circ \{\!|C|\!\} \\
\therefore\ P(F_w f, F_r g) &= \forall C, E(F_w f) \circ \{\!|C|\!\} = F_r g
\end{aligned}
$$

□

Exercise 2 *P($fix(Fx)$, $fix(liftF)$) ≜ ∀x ∈ X ∀a ∈ A, ($fix(Fx)$) a = ($fix(liftF)$) ⟨x, a⟩*

Proof.
P는 품에 넣는 성질(inclusive assertion)을 만족시키므로, 고정점 귀납법으로 P(⊥, ⊥) 이고, P(f,g) ⇒ P((F x) f, (lift F) g) 임을 보이면 된다.

1) P(⊥, ⊥)

$$\forall x \in X, \forall a \in A\ \perp_{A\rightarrow B} a = \perp_{(X\times A)\rightarrow B}\langle x, a\rangle = \perp_B$$

2) P(f,g) ⇒ P((F x) f, (lift F) g)

$$P(f,g) = \forall x \in X \ \forall a \in A,\ f\ a = g\langle x,a\rangle \tag{1}$$

임을 가정하고,

$$P((F\ x)f,(lift\ F)g) = \forall x \in X, \forall a \in A\ ((F\ x)f)a = ((lift\ F)g)\langle x,a\rangle \tag{2}$$

을 보인다.

$$\begin{aligned} ((lift\ F)g)\langle x,a\rangle &= ((F\ x)(\lambda a'.g\langle x,a'\rangle))a \\ &= ((F\ x)f)a\ (\because \forall x \in X, f = \lambda a'.g\langle x,a'\rangle\ by\ (1)) \end{aligned}$$

$$\therefore P((F\ x)f,(lift\ F)g).$$

□